

# Chapter 13 — Solving Proportions with More Than One Strategy — Teacher Guide

## Chapter at a glance

|                                     |                                                                                                                                                                                                                                                                                                                                                    |
|-------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Lesson duration</b>              | 5 lessons, 40–65 minutes each (see pacing table below)                                                                                                                                                                                                                                                                                             |
| <b>Materials</b>                    | Two-color counters, ratio cards, tape strips, ratio tables, unit-rate cards, equation cards, cross-product arrows, map-scale cards, “proportional or not?” situation cards, the two chapter diagrams                                                                                                                                               |
| <b>Launch question</b>              | “A recipe uses 2 cups of flour for 5 batches. How many cups for 20 batches — and how many different ways can you solve it?”                                                                                                                                                                                                                        |
| <b>Key representation</b>           | The four-method decision map — keep it visible throughout instruction                                                                                                                                                                                                                                                                              |
| <b>Anticipated responses</b>        | Correct: preserves order, picks a method suited to the numbers, and checks the answer against the original ratio. Partial: solves correctly but can’t explain why that method fit, or checks by repeating the same steps. Incorrect: adds instead of multiplies, reverses cross-products, or applies a proportion to a non-proportional situation. |
| <b>If the readiness check fails</b> | Reteach equivalent ratios and unit rates from Chapters 10 and 11 before starting the Launch; do not skip collecting multiple student methods before naming them                                                                                                                                                                                    |
| <b>Talk prompt</b>                  | “What does each number represent, and what factor connects the known pair?”                                                                                                                                                                                                                                                                        |
| <b>Exit ticket</b>                  | 5 items — solve by scale factor, solve by cross-products, apply to a context, choose a method and justify it, explain a non-proportional situation (see Section 14)                                                                                                                                                                                |
| <b>Reteach / extend</b>             | Reteach: scale-factor and unit-rate methods only, with concrete equal groups. Extend: decimal solutions, non-proportional counterexamples, comparing all four methods on one problem.                                                                                                                                                              |

## Purpose and pacing

Chapter 13 brings together the ratio, rate, and comparison ideas from Chapters 9–12. The goal is not to make every student use every method equally. The goal is for students to understand the shared relationship and choose a method deliberately.

| Lesson   | Focus                                                     | Suggested time |
|----------|-----------------------------------------------------------|----------------|
| Lesson 1 | Readiness, launch, scale factor, and ratio tables         | 50–65 minutes  |
| Lesson 2 | Unit rate and representation connections                  | 45–60 minutes  |
| Lesson 3 | Cross-products, order, and misconception repair           | 50–65 minutes  |
| Lesson 4 | Non-proportional situations, mixed practice, and Tripwire | 50–65 minutes  |
| Lesson 5 | Exit Ticket, extensions, and targeted reteaching          | 40–55 minutes  |

## Materials

Prepare two-color counters, ratio cards, tape strips, ratio tables, unit-rate cards, equation cards, cross-product arrows, map-scale cards, and “proportional or not?” situation cards.

## Teacher launch guidance

Begin with the recipe question and ask students to solve it in more than one way. Collect methods before naming them. This prevents the chapter from becoming a procedure lesson and makes the common multiplicative structure visible.

When introducing cross-products, connect the operation to the equality of ratios. Students should know what each ratio measures before writing the equation. When a student reverses one ratio, return to labels rather than simply correcting the symbols.

The non-proportional example is essential. Use a taxi with a starting fee or a gym with a membership charge. Ask what the output would be at zero input. A direct proportion must pass through zero.

## Teacher prompts

| Purpose               | Prompt                                                  |
|-----------------------|---------------------------------------------------------|
| Identify relationship | “What quantities are being compared?”                   |
| Preserve order        | “What does the numerator measure in both ratios?”       |
| Find scale            | “What factor changes the known value?”                  |
| Use a ratio table     | “Can you build rows without changing the relationship?” |
| Use unit rate         | “Would finding one unit make the target easy?”          |

| Purpose              | Prompt                                                      |
|----------------------|-------------------------------------------------------------|
| Use cross-products   | "Is the scale factor awkward but the ratio equation clear?" |
| Check                | "Does the answer give the same ratio as the original?"      |
| Test proportionality | "What happens when the input is zero?"                      |

## Differentiation

### Students needing concrete support

Use two-color ratio groups and physically copy groups to show a scale factor. Keep numerator and denominator labels attached to the counters. Move to tables only after students can explain why both quantities must be scaled by the same factor.

### Students needing language support

Use the frames:

- "The first ratio compares \_\_\_\_\_ with \_\_\_\_\_."
- "The scale factor is \_\_\_\_\_ because \_\_\_\_\_."
- "I chose \_\_\_\_\_ because \_\_\_\_\_."
- "The ratio order stays the same: \_\_\_\_\_ over \_\_\_\_\_."
- "I checked by \_\_\_\_\_."

### Students ready to extend

Require two methods for the same problem, ask students to construct non-proportional counterexamples, and include decimal solutions and unit conversions.

### Formative assessment

Record whether students identify quantities, preserve order, recognize common scale factors, use unit rates, execute cross-products, divide to isolate the unknown, test proportionality, and justify method choice. A correct answer without a preserved relationship should be marked developing.

### Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Solve  $y = x/20$  using a scale factor. Solve  $y = 12/x$  and state the unknown. Explain when cross multiplication is efficient but not explanatory by itself. Check a proportion with an estimate or equivalent ratio.

Strategy Studio guidance: Expected ideas:  $x = 12$ ; 4:6 and 10:15 are both 2:3; in  $y = 8/x$ ,  $x = 12$ ; students should preserve the same multiplicative factor or use equivalent products. Treat method comparison as mathematical communication, not merely preference.

### Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error.

Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: scaled by 4 gives  $x = 12$ . Altered:  $= 8/x$  gives  $2x = 24$  and  $x = 12$ . Non-routine: 4:6 and 10:15 both simplify to 2:3. Repair: cross multiplication preserves equality because equivalent fractions have equal cross-products; the representation should still show the proportional relationship.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

## Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

## Answer Key

### Ready-to-Learn Check

1. Same relationship.
2. 10.
3. \$3/notebook.
4. 240 km.
5. Order carries the units and meaning.
6. 4.
7. No; equivalent ratios require one common multiplier.

### Worksheet A

1. **2:5 = 4:10, so  $x = 10$ .** Likely wrong answer: 7 (adds 2 to 5 instead of multiplying, since 2 changed to 4 by adding 2). Misconception: additive rather than multiplicative scaling. Next move: "what factor — not what amount added — changes 2 into 4?"
2. **3:7 = 12:28, so  $x = 28$ .** Likely wrong answer: 16 (adds 9 to 7, mirroring the additive jump from 3 to 12). Misconception: same as #1. Next move: same check.
3. **Scale by 3, so  $x = 12$ .** Likely wrong answer: 22 (adds 18 to 4, mirroring the additive jump from 9 to 27). Misconception: same as #1. Next move: same check.
4. **\$3/pen  $\times$  7 = \$21.** Likely wrong answer: \$16 (adds 12 and 4 instead of finding the unit rate and multiplying). Misconception: confusing scaling with combining. Next move: "what does one pen cost? Multiply that by 7."
5. **60 km/h  $\times$  4 = 240 km.** Likely wrong answer: 184 km (adds 180 and 4, mixing distance and time units together). Misconception: same as #4. Next move: "find the speed first, then multiply by the new time."
6.  **$x = 15$ .** Likely wrong answer: cross-multiplies incorrectly, multiplying the two numerators or two denominators together instead of the diagonal pairs. Misconception: incorrect cross-products. Next move: "which two numbers form each diagonal in the equation?"
7. **It changes the units and meaning.** Common flaw: says "it's wrong" without explaining that the numerator and denominator now measure different things. Next move: ask what the numerator represents in each version.
8. **Unit rate is efficient because \$12  $\div$  4 = \$3/pen.** Common flaw: picks

a method without explaining why it fits these particular numbers. Next move: ask what makes this problem well-suited to a unit rate versus a scale factor.

9. **Rows: 1 h/60 km, 3 h/180 km, 4 h/240 km or equivalent table.** Common flaw: builds a table with rows that don't share a consistent scale factor. Next move: check each row against the one before it.
10. **Answers vary but must agree.** Common flaw: uses a second method but arrives at a different answer without noticing or reconciling the mismatch. Next move: "do both methods give the same value of  $x$ ?"

## Worksheet B

11.  **$=$ , so  $3x = 48$ ,  $x = 16$ .** Likely wrong answer: cross-multiplies the wrong pairs, such as  $3 \times 12 = 4x$ , giving  $x = 9$ . Misconception: incorrect cross-products — multiplying two numbers that aren't diagonal to each other. Next move: "which two numbers are diagonal from each other in this equation?"
12.  **$8x = 120$ ,  $x = 15$ .** Likely wrong answer: stops at  $8x = 120$  without dividing to isolate  $x$ . Misconception: fails to complete the final step. Next move: "what operation undoes multiplying by 8?"
13.  **$7x = 210$ ,  $x = 30$ .** Likely wrong answer: same incomplete-division error as #2. Misconception: same as #2. Next move: same check.
14. **Scale factor 5 ( $20 \div 4 = 5$ ), so scale 9 by 5:  $x = 45$ .** Likely wrong answer: 36 (uses the wrong scale factor, mistakenly applying 4 instead of the correct factor 5). Misconception: miscalculating the scale factor by not dividing the corresponding known pair correctly.
- Next move: "what number multiplies 4 to give 20? Use that same number on 9."
15.  **$\$3/\text{marker} \times 9 = \$27$ .** Likely wrong answer: \$24 (adds 15 and 9 instead of finding the unit rate and multiplying). Misconception: confusing scaling with combining. Next move: "what does one marker cost? Multiply that by 9."
16.  **$24 \text{ km/h} \times 7 = 168 \text{ km}$ .** Likely wrong answer: 103 km (adds 96 and 7, mixing distance and time units together). Misconception: same as #5. Next move: "find the speed first, then multiply by the new time."
17.  **$25 \text{ km/cm} \times 8 = 200 \text{ km}$ .** Likely wrong answer: 83 km (adds 75 and 8 instead of finding the unit rate and multiplying). Misconception: same as #5. Next move: same check.
18.  **$= 0.6$ ,  $= 0.5$ ; not equal.** Likely wrong answer: says they're equal because both "have a 5 in them," without converting either to a decimal or checking a common scale factor. Misconception: judging equivalence by appearance instead of computing. Next move: "convert both fractions to decimals and compare."
19. **Answers vary but must be correct.** Common flaw: the unit-rate solution and the cross-products solution give different final answers, and the mismatch goes unnoticed. Next move: "do both methods agree?"
20. **Answers vary; must identify a nonzero starting amount or changing rate.** Common flaw: describes a situation as "not proportional" without identifying the specific starting amount or changing rate that breaks proportionality.

Next move: “what is the output when the input is zero?”

## Worksheet C

1. **cup per muffin  $\times$  24 = 15 cups.** Likely wrong answer: 29 cups (adds 5 and 24 instead of finding the unit rate and multiplying, or scales incorrectly). Misconception: confusing scaling with combining. Next move: “how much flour does one muffin need? Multiply by 24.”
2. **\$13/h  $\times$  13 = \$169.** Likely wrong answer: \$117 (adds 104 and 13 instead of finding the unit rate and multiplying). Misconception: confusing scaling with combining. Next move: “what does the worker earn per hour? Multiply by 13.”
3. **63 km/h  $\times$  8 = 504 km.** Likely wrong answer: 320 km (adds 315 and 5 instead of finding the unit rate and multiplying). Misconception: same as #1. Next move: same check.
4. **12x = 210, x = 17.5.** Likely wrong answer: cross-multiplies the wrong pairs (for example,  $7 \times 12$  instead of  $7 \times 30$ ). Misconception: incorrect cross-products. Next move: “which two numbers are diagonal from each other?”
5. **= ; 45 = 3x; x = 15.** Likely wrong answer: sets up the proportion with the terms reversed, giving a different value of x. Misconception: switched order — not matching which quantity is the numerator on each side. Next move: “does 9 correspond to the 3, or to the 5?”
6. **Scale factor 5 because 7 to 35 is  $\times 5$ , so x = 25.** Likely wrong answer: 30 (adds 28 to 5 instead of multiplying by the scale factor). Misconception: additive rather than multiplicative scaling. Next move: “what factor — not what amount added — changes 7 into 35?”
7. **Other methods may show units and meaning more clearly.** Common flaw: says cross-products are “harder” without explaining that the specific weakness is losing sight of what the numbers represent. Next move: ask what a ratio table shows that cross-products don’t.
8. **Answers vary but must solve two consistent ways.** Common flaw: the two methods give different final answers, and the mismatch isn’t noticed or resolved. Next move: “do both methods agree?”
9. **The starting fee means cost is not zero at zero distance.** Common flaw: says “it’s not proportional” without identifying the specific zero-input test that shows why. Next move: “what would the cost be at 0 km? Should it be \$0 for a direct proportion?”
10. **Must include labels, equation, and inverse check.** Common flaw: gives a correct numeric answer without the supporting diagram, equation, or check. Next move: require all three components before accepting the answer.
11. **20 km. The daily training time (2 hours) and the weekly schedule (5 days) were not needed to find the distance.** Likely wrong answer: works the 2 hours or 5 days into the calculation somehow. Misconception: assuming every number in a word problem must be used. Next move: “what does the question actually ask for? Which numbers answer that?”
12. **Answers vary; must describe a situation where 6 is a known quantity, 210 is a total or target amount, and 35 is the unknown rate or scale factor, with each meaning stated.** Common flaw: gives a context but doesn’t identify all three roles, or confuses which

*number is the total and which is the rate. Next move: ask the student to finish three sentences — “6 is \_\_,” “210 is \_\_,” “35 is \_\_.”*

### Extend Your Thinking

1.  $\approx 0.583$ ;  $x/30 \approx 0.583$ , so  $x = 17.5$ ; cross-products also give  $12x = 210$ .
2.  $=$ , so  $x = 15$ .
3. Answers vary but must justify the decimal.
4.  $=$ ;  $x = 30$ .
5. Answers vary but must include a nonzero starting amount or changing multiplier.

6. Cross-products are efficient when scale is unclear; tables are more informative when patterns matter.

### Tripwire and Exit Ticket

1. Tripwire: 3:5 is not equivalent to 6:8; correct pair is 6:10.

**Exit Ticket:**  $x = 8$ ;

$x = 17.5$ ;

7.5 cups; answers vary but must justify method choice;

the starting fee means output is not zero when input is zero.

